**Electrodynamics in Metal-Insulators**

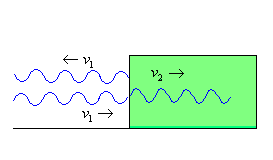
Now let’s work out some actual examples. First we’ll restate our equations:

And then,

**Transmission through semi-infinite metallic interface (normal incidence)**

Now let’s consider the case of normal incidence on a metallic interface in 1D. Suppose we have an **E**-field impinging upon a substance from the left (this field we would presume to know). There will be a reflected and transmitted wave so…



**LHS Solution**

On the left hand side we have,



And our solution would be, say



Plugging this into the equation we find that,



and also:



where **k**1± = ±.

**RHS Solution**

And on the right we’ll have:





Plugging into the equation yields:



and also:



as usual.

**Boundary Conditions**

Now we need to enforce the boundary conditions. There is no free charge after the time constant. And there is no free current at the surface. For instance, consider a nearly hollow cylinder with thickness Δd. The current running through the thickness would be given by **j** = σ**E** by assumption. As we shrink the thickness Δd→0, the current running through the surface would also go to 0. And thus there is no surface current when the current is given by Ohm’s law. So our boundary conditions are:



Now match up boundary conditions. The first is automatically satisfied since **E** is a transverse wave. Same with the third condition on **B**. The second requires:



which just means that **E** must be continuous across the boundary. The fourth requires:



Plugging in the relation between **E** and **B** we get,



(we remember that **k**1- points back to the left and define the complex velocity v = ω/k, where k is the complex wavevector). So our two equations are:



**Reflection/Transmission Amplitudes**

Now we can define the reflection and transmission amplitudes, just like we do for QM. And we’ll be defining these quantities at the r = 0 interface between the two media.



Written in these terms, our equations become:



These can be rearranged to:



with solutions,



where we have defined β = n2/n1 as usual, and where in the ≈ step we used the fact that μ ≈ μ0 for most materials. Let’s work these out, under the assumption that the material is a good conductor, and that μ ≈ μ0, etc.



And the reflection coefficient is r = t – 1 (as can see from the formula above). So we’ll note that if the conductor is very good σ → ∞, then r → -1 which corresponds to complete reflection (and inversion) of the wave.

**Transmission/Reflection Coefficients**

And now, the actual transmittance, and reflectance is proportional to the intensity that makes it through.



Now, **S** = **E**×**H**, so we have to form **S** in each medium, relate to **E**, and then take the ratio, and form in terms of *t* and *r* (can use <S> = (1/2)Re(E\*H) = (1/2)Re(EH\*)). We’ll presume μ1 = μ2 ≈ μ0. We already have a formula for <S> from the previous file, but I’ll just start from here anyway.



and similarly,



where again we use the fact that μ ≈ μ0 for all materials and thus the distinction between H and B for different materials collapses. Let’s work it out for the case when the material is a good conductor. First, since n1 is real (I’m assuming), we have:



So plugging this in:



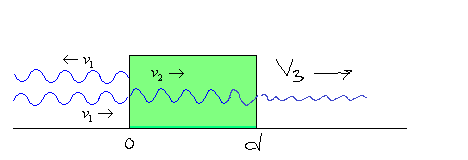
So the transmission and reflection coefficients would be approximately:



So transmission increases with frequency, as is known, and decreases with conductivity, which makes sense too. And if we have a perfect conductor (σ = ∞), then we get complete reflection. Of course T + R = 1, by energy conservation. But in example below, we shouldn’t have this equality, i.e., T = <S3+>/<S1+> and R = <S1->/<S1+> shouldn’t add up to 1, because of energy absorption in the interim, within the finite width metal block.

**Transmission through finite metallic interface (normal incidence)**

Now let’s consider the case of normal incidence on a metallic interface in 1D. Suppose we have an **E**-field impinging upon a substance from the left (again, which we’d presume to know). There will be a reflected and transmitted wave so…



**LHS Solution**

On the left hand side we have,



And our solution would be, say



Plugging this into the equation we find that,



and also:



**Middle Solution**

And in the middle we’ll have:





Plugging into the equation yields:



and also:



as usual.

**Enforcing LHS/Middle Boundary Conditions**

Now we need to enforce the boundary conditions. There is no free charge after the time constant. And there is no free current at the surface. For instance, consider a nearly hollow cylinder with thickness Δd. The current running through the thickness would be given by **j** = σ**E** by assumption. As we shrink the thickness Δd→0, the current running through the surface would also go to 0. And thus there is no surface current when the current is given by Ohm’s law. So our boundary conditions are:



Now match up boundary conditions. The first is automatically satisfied since **E** is a transverse wave. Same with the third condition on **B**. The second requires:



which just means that **E** must be continuous across the boundary. The fourth requires:



Plugging in the relation between **E** and **B** we get,



(we remember that **k**1- points back to the left and define the complex velocity ). So our two equations are:



and now for the last interface.

**RHS Solution**

At the right we’ll have:





Plugging into the equation yields:



and also:



**Middle/RHS Boundary Conditions**

and boundary conditions at x = d will be:



and the last requires:



So our two equations are:



**Transmission/Reflection Amplitudes**

and all together we have:



Now we’d like to solve for the amplitudes in terms of the free variable E1+. I suppose I’ll organize it as a matrix:



and so then,



where f12 and f32 are obvious definitions. Note that since the μ’s will be approximately the same, I’ll have that approximately f12 = n2/n1, and f32 = n2/n3.

Now I’ve got to evaluate the inverse. Well, all I really want are E1- and E3+. So let me just eliminate the other variables. Solving for E2+ and plugging in I’ll get



Simplifying,



Now solve for E2- and substitute in:



Arranging nice and pretty like:



general solution of



So then we have for r and t,



The numerator of t can be simplified a bit:



and so we have:



If n3 = n2, then we ought to recover the previous result. In this case we’d have f32 = 1, and k3 = k2. So let’s see…



which is correct!

**Transmission/Reflection Coefficients**

And now, the actual transmittance, and reflectance is proportional to the intensity that makes it through.



and this reduces to:



where again we use the fact that μ ≈ μ0 for all materials and thus the distinction between H and B for different materials collapses. Therefore we have, utilizing fact that n3/n1 = f13, and f13∙f312 = f31,



or in other words:



Of course in my example, I have n1 = n3.



Let’s fill this into the formula, and I’ll just call n2, n.



Continuing,



and some more:



Simplifying a little,



and,



and it appears this is the best we can do. Wow that’s nasty.



What is the limit as ω → 0? Then kR/I goes to equal constants. And nR/I goes to equal 1/ω dependent constants. So filling this in,



Suppose also that σ → ∞. Then what? Then the k’s become infinite, and n’s do too. Then we get, it seems,



and this definitely goes to 0, no matter the d. This can be ascribed to the energy absorption within the block. Would have to get R separately, therefore, as we cannot say R = 1 – T.