**Form of the Dipole-Dipole Interaction**

So I want to work out the form of the dipole-dipole interaction, for both cases: electric and magnetic. Seems like this is a good place to put it. So,

**Electric Dipole-Dipole Interaction**

So it would take the form,



where ρb is the bound charge density. Recal that the bound charge density is ρ­b(**r**) = -∇·**P**(r), where **P**(r) is the polarization density. Working this out,



where we use ∇**r** = **1**, the unit tensor, and a lot of integration by parts. So,



What if the polarization is uniform? Then it clearly simplifies, but don’t want to start from this formula. We know we’ll basically just have surface bound charge, and so our box formula will just reduce to a surface integral. But still, we know that in any case, this is just:



And Eb = σb/ε0 = P/ε0, at least for a capacitor-like situation. So then we have:



where ΔV is volume of our dielectric.

**Magnetic Dipole-Dipole Interaction**

So,



(see Energy Thoughts file for a derivation of this form) And recall that the bound current density is **j**b(r) = ∇×**M**(r), where **M**(r) is the magnetization density. So working this out:



Now use,



In our situation, only the gradient acting on **A** survives, so this effectively reduces to:



and we get:



Do we ignore the δ guy? Seen some keep and some discard. I’m discarding….so,



What if the magnetization is uniform? Then it clearly simplifies, but don’t want to start from this formula. We know we’ll basically just have bound surface current, and so our box formula will just reduce to a surface integral. But still, we know that in any case, this is just:



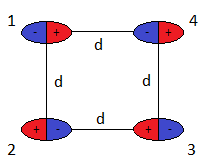
And **B**b = μ0**K**b, where K is bound surface current, and **K**b = ×**M**. At least for solenoid-like situation. So then we have:



where ΔV is volume of our dimagnet.

**Example**

Say we had something like the ‘molecule’ below. Let each dipole have a magnitude P. What is the potential energy of the configuration, to within the dipole approximation?



Let’s label the dipoles counterclockwise starting from top left as 1, 2, 3, 4.



which is,



which is,



and is,



Finally,



Really finally,



So this is clearly negative, which indicates that the dipoles would prefer to be bound together in this arrangement, than independently floating separately. But, we didn’t demonstrate that this arrangment has the lowest potential energy. And also didn’t address the question of why d shouldn’t go to zero, which would ostensibly reduce the potential energy to -∞. Well, there would be repulsion between them when they get too close, and this is where the dipole approximation breaks down. So to find the d which would minimize the potential energy in this arrangement, we’d have to go beyond the dipole approximation.